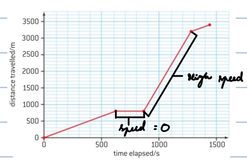
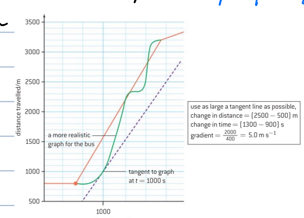
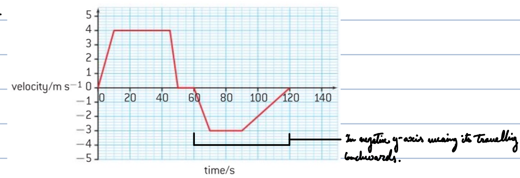
- 2.1 Motion
  - Equations
    - $v = u + at$
    - $s = ut + \frac{1}{2}at^2$
    - $v^2 = u^2 + 2as$
    - $s = \frac{(v+u)t}{2}$
  - Distance and displacement
    - Distance is a scalar quantity, as it has no direction.
    - Journeys can be defined by the starting point and its finishing point.
      - The vector measurement of distance has direction and magnitude, and is known as displacement.
        - The distance & magnitude have their own units.
      - Distance and displacement aren't the same thing even though they might have the same value.
        - Displacement has the definition: the distance from your initial position to your final one.
        - Distance has the definition: the distance travelled in a given amount of time.
  - Speed and Velocity
    - Velocity is the vector of speed.
      - Speed has the definition: $\frac{\text{distance travelled}}{\text{time taken for journey}}$.
        - The units for speed are "ms^{-1}".
  - Worked example
    - 16km in 70 mins: $\frac{16000}{70 \cdot 60} = 3.81\ ms^{-1}$
    - $3 \cdot 10^8 \cdot 365 \cdot 24 \cdot 60 \cdot 60 = 9.5 \cdot 10^{15}\ m$
      - $22 \cdot 9.5 \cdot 10^{15} = \frac{2.1 \cdot 10^{17}\ m}{1000} = 2.1 \cdot 10^{14}\ km$
  - Describing motion with a graph
    - Since its unlikely that the a person will travel at the same velocity throughout the entire journey.
      - This can be seen in the graph below:
        - 

        - The straight line section shows that there was no displacement as there is no change in the the y-axis.
          - The steeper the graph the faster the person is moving.
        - The gradient shows the speed, adding directions will give velocity
  - Instantaneous and average values
    - The instantaneous speed is the speed of the object at a precise moment in time.
    - 

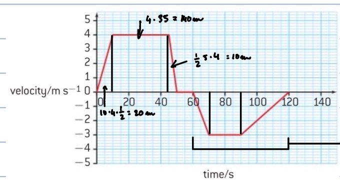
      - The instantaneous speed can be described as the rate of change position with respect to time. ($\frac{\partial s}{\partial t}$)
        - $\frac{\partial s}{\partial t} = \frac{\text{change in distance}}{\text{change in time}}$
    - The average speed is given by: average speed = $\frac{\text{distance travelled in all the journey}}{\text{time for the whole journey}}$
      - The average speed is usually a straight line.
    - What is said about the average and instantaneous speeds all translates to average and instantaneous velocities.
  - Worked example (33)
    - i) $\frac{48}{10} = 4.8\ ms^{-1}$ ii) $\frac{90-48}{20} = 2.1\ ms^{-1}$
    - $3\ ms^{-1}$

- Acceleration
    - Acceleration is a vector quantity.
        - The definition is: acceleration $= \frac{\text{change in velocity}}{\text{time taken for the change}}$.
        - Units = $m s^{-2}$.
    - If an object has an acceleration of $5 m s^{-1}$, then every second its velocity will increase by $5 m s^{-1}$.
- Worked example
    - Acceleration $= 0.72 m s^{-2}$ $\quad$ $u = 300 km h^{-1} \equiv 83.3 m s^{-1}$ $\left(\frac{300}{3.6} = 83.\overline{33}\right)$

      $\frac{83.3}{0.72} = 115.7$ seconds $\simeq$ 2 mins
- Describing motion with a graph II
    -


▲ Figure 6 Velocity–time graph for the bicycle.

- The area under the graph gives the displacement.
    - To find the area, calculus can be used if the equation of the graph is given, or the area formulas for squares, triangles, and rectangles can be used.
- The gradient of the graph gives the magnitude of acceleration.
    - The change in velocity is given by: $\frac{\text{(final velocity)} - \text{(initial velocity)}}{\text{time taken}}$.

- E.g


▲ Figure 6 Velocity–time graph for the bicycle.

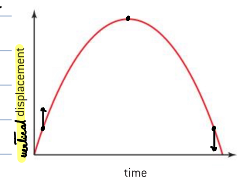
- When the graph is curved you need to:
    - Estimate the number of squares
    - Assess the area (distance) for one square
    - multiply the number of squares by the area of a square
- suvat equations
    -

symbol	quantity
s	displacement/distance
u	initial (starting) velocity/speed
v	final velocity/speed
a	acceleration
t	time taken to travel the distance *s*

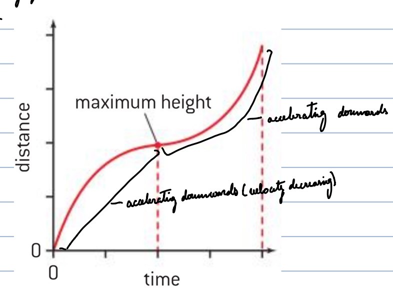
- $v = u + at$ (first equation of motion)
- $s = ut + \frac{1}{2}at^2$ (second equation of motion)
- $v^2 = u^2 + 2as$ (third equation of motion)
- $s = \left(\frac{u+v}{2}\right)t$ (fourth equation of motion)
- Worked example
    - $150 = \frac{25}{2}t$ $\quad$ $\frac{300}{25} = t$ $\quad$ $t = 12 s$ ✓
    - $-\frac{25}{12} = a$ $\quad$ $a = -2.1 m s^{-2}$ ✓
    - $\frac{2.5 - 7.5}{5} = a$ $\quad$ $a = -1 m s^{-2}$ ✓
    - $s = (7.5 \cdot 5) + \frac{1}{2}(-1)(25)$ $\quad$ $= 25 m$ ✓
- Projectile motion
    - Falling freely
        - When an object is released near the surface of the Earth, it accelerates downwards.
            - Due to gravity pulling it down. Given by "g".

- The object will also pull the Earth with the same force.
    - We don't notice because the object is so small.
- The value for gravity changes in different parts of the world. This is due to the densities of the rocks in different locations.

- What goes up must come down
    - After a ball has been thrown in the air the gravity will instantly take effect on the object instantly.
        - The ball will then start slowing down, as acceleration is down due to gravity.
    - If there is no air resistance, then the horizontal displacement per second won't change.
        - Example graph:



- As seen in the graph, the object is simply going up, and then coming back down in its same place. If the object has a horizontal displacement then it will have the same trajectory shape of the graph (if there is no air resistance), but if the ball is shot into the air from an moving object, it will land in the same place (if no air resistance) because it will have the same horizontal displacement as the object that its shots out of.
    - A distance-time graph instead shows the distance travelled by the ball, not the displacement.
        - Example graph:



- The gradient shows the speed of the ball at a particular time.
- The object will always be accelerating downwards due to gravity.

- Worked examples
    - $t = 2.3s$, $v = ?$, $s = ?$, $u = 0$, $a = 9.81ms^{-2}$ – $v = \sqrt{u^2 + 2as}$ – The speed of sound is $300ms^{-1}$ so for something which is 12 times less than the speed of sound can be ignored ✓

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(9.81)(2.3)^2$

$= 25.95m$

$\approx 26.0m$ ✓

$= \sqrt{2(9.81)(25.95)}$

$v = 22.563 ms^{-1}$

$\approx 22.6 ms^{-1}$ ✓

- $a = 0ms^{-2}$,

$s = ?$, $v = 0$, $u = 5.0ms^{-2}$, $a = 9.81ms^{-2}$, $t = 0.51s$

$v = u + at$

$\frac{-5}{-9.81} = t$

$t = 0.509683995$

$\approx 0.51s$

$s = ut + \frac{1}{2}at^2$

$= 5 \cdot 0.51 + \frac{1}{2}(-9.81)(0.51)^2$

$= 1.274$

max displacement $= 31.27m$

- 0.51s

- $s = -31.27m$, $v = 0ms^{-1}$, $u = 0ms^{-1}$, $a = -9.81ms^{-2}$, $t ?$

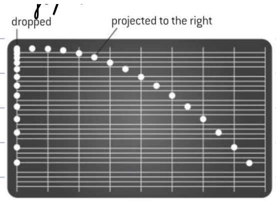
$\sqrt{\frac{-31.27(2)}{-9.81}} = t$

$t = 2.53s$

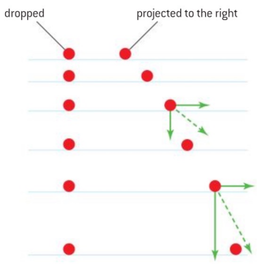
$2.53 + 0.51 = 3.04s$

- Moving horizontally
    - Gravity acts vertically and not in the horizontal direction. Therefore, the horizontal motions are:
        - The horizontal velocity doesn't change
        - The horizontal distance travelled is horizontal speed × time for motion.

- Putting it all together
    - If a ball is thrown horizontally it will have the same vertical displacement of a ball that was just dropped.
        - As seen in the image below, the change vertically is the same.
        - The horizontal displacement also doesn't change if air resistance is negligible.
        - The graph also shows that in the vertical plane displacement and time have the relationship $s \propto t^2$.

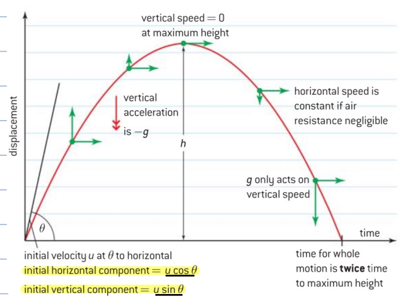


▲ Figure 13 Multi-flash images of two falling objects.



▲ Figure 14 Horizontal and vertical speed components.

- the two motions, horizontal and vertical, are completely independent of one another.
  - horizontal speed continues unchanged.
  - vertical speed will change due to gravity.
-



at maximum height:

$0 = u\underbrace{\sin\theta} - gt$

$h = (u\underbrace{\sin\theta})t - \frac{gt^2}{2}$ and

$0 = u^2\underbrace{\sin^2\theta}_{\text{vertical component}} - 2gh$

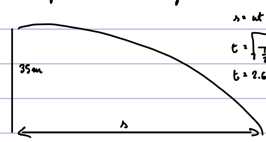
range:

$= 2t\,(u\underbrace{\cos\theta}_{\text{horizontal component}})$

t is the time to maximum height

  - The reason that cos and sin are used is because of the fact that the ball was shot at an angle.

- worked example
  - 35m from the ground, horizontal velocity = $30ms^{-1}$
    -



$s = ut + \frac{1}{2}at^2$

$t = \sqrt{\frac{35}{\frac{1}{2}(9.81)}}$

$t = 2.67s$ ✓

- $s = ut$
  $s = (2.67)(30)$
  $s = 80.14m$
  $\approx 80m$ ✓

- $30ms^{-1}$ (horizontal) + (vertical) $v = \sqrt{u^2 + 2as}$
  $= \sqrt{2(9.81)(35)}$
  $\underline{v = 26.2ms^{-1}}$
  $\sqrt{30^2 + 26.2^2} = \underline{39.8ms^{-1}}$
  total final velocity = $\underline{39.8ms^{-1}}$

Final velocity = $\sqrt{\text{horizontal velocity}^2 + \text{vertical speed}^2}$ ≠ horizontal speed + vertical speed

this is the final velocity of an object shot at an angle, like an arrow.

- $s = \frac{(v+u)t}{2}$
  $\frac{2s}{t} = 2u$
  $u = \frac{2(37)}{2(1.6)}$
  $u = 23.125$
  $\approx 23.1ms^{-1}$

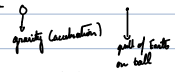
- $s = ut + \frac{1}{2}at^2$
  $s = \frac{1}{2}(9.81)(1.6)^2$
  $\underline{s = 12.6m}$

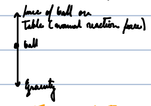
- 2.2 Forces
  - formulas
    - Newton's second law: $F = ma$
    - static friction equation: $F_f \le \mu_s R$
    - dynamic friction equation: $F_f = \mu_d R$
  - Newton's laws of motion

- Newton's first law
  - Moving objects have inertia, meaning a resistance to stopping and that, once in motion, objects continue to move.
  - Assuming that no energy is lost, the ball if put in a system where it rolls up and down a slope () the ball will never stop, and will always return to its max displacement from the equilibrium point.
    - The object, if it remains uninterrupted, will move on forever if it can't reach its vertical position again.
    - The object will continue until it reaches its vertical position again.
  - Newton's first law says:
    - An object continues in motion or remain stationary unless an external force acts on it.
    - Unless something like friction or an applied force acts on an object the object will remain in its state indefinitely.
      - Both speed and direction must remain the same.
- Newton's second law
  - The second law of motion can be written in two different ways. The first one is:
    - Force = mass · acceleration → F=ma.
    - Units are newtons, or $kg\,m\,s^{-2}$.
  - Two things come from this equation:
    - Mass is a scalar, so there will be no change in direction of the acceleration if acceleration is multiplied by mass.
      - The direction of the force is the same as the acceleration's direction as acceleration is the only vector quantity, meaning direction and magnitude are dictated by the acceleration & not mass.
      - So applying a force to a mass will change the velocity in the same direction as that of the force.
    - One way to think about the mass in this equation is that it is the ratio of the force required per unit of acceleration for a given object.
      - If the mass is 1kg and the acceleration is $1\,m\,s^{-1}$, then one unit of force (N) must have acted on it.
- Worked example
  - mass = 1500 kg, $a = 28\,m\,s^{-1}$, t = 11s.
    - $F = \frac{1500 \cdot 28}{11}$
      $F = 3818.2\,N$
      $\approx \underline{3800N}$ ✓

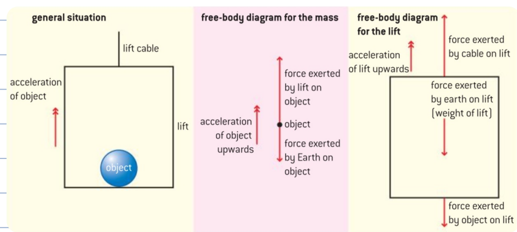
  - $m = 3.3 \cdot 10^5\,kg$, $s = 1.7\,km$, $F = 830\,kN$
    - $a = \frac{830000}{3.3 \cdot 10^5}$ $= 2.52\,m\,s^{-2}$
      $v^2 = u^2 + 2as$
      $v = \sqrt{2as}$
      $v = 92.5\,m\,s^{-1}$ ✓
- Newton's third law
  - Newton's third law of motion can be written as:
    - For every action there is an equal and opposite reaction.
  - The law suggests that forces appear in pairs.
    - E.g ball on a table, gravity down, and normal force will be the reaction force.
      - In reality the table will very slightly be dented, meaning that the ball will also slightly deform.
      - The reason that the table will be dented is because of the fact that the gravitational force will bring the ball down, pushing the atoms of the ball and the table closer together, resulting in a resistance force to the force down.
        - The ball flattening is also a reaction force to the pull downwards
    - Example of rocket propulsion
      - Inside the rocket chemicals react together producing a gas with a very high temperature and pressure.
      - The rocket has exhaust nozzles where the gas escapes from the combustion chamber.
      - At one end of the chamber the gas molecules rebound off the wall to exert a force, reversing their direction.
        - They then travel down through the nozzel so that they can't exert a force on the back of the rocket cancelling out the forward force generated.
- Free-body force diagrams
  - Vectors can be represented by arrows, with scaled length and direction.
  - The rules for free body diagrams is:
    - One body only and force vectors are represented by arrows.
    - Only forces acting on body are considered
    - The force arrows originate from the center of mass
    - Must have a label.

- Examples

  - 

  - 

- The object accelerating in an elevator

  - The weight of the object is downwards and the size of this force is the same as though the object had been stationary on Earth's surface.
  - However, as the elevator starts moving up, for a split second the weight will be greater due to the fact that the inertia has to be overcome. Therefore, the increase in the acceleration upwards will result in an increase in weight as the inertia has to be overcome.
    - The opposite will occur when the elevator is moving down. For a split second the weight of the person / object will be less.
  - 

    - The majority of the force will be felt on the cable.

- Translation equilibrium

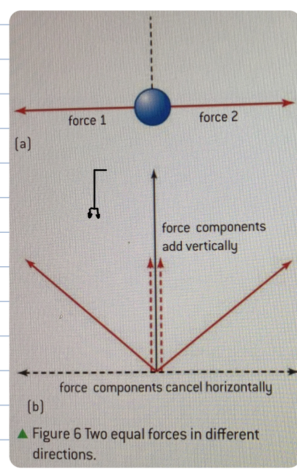
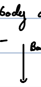
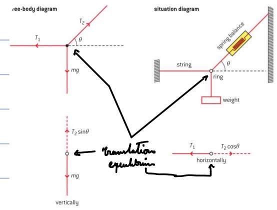
  - When an object is at translation equilibrium is either at rest or moving at constant velocity.
    - Translation means moving in a straight line.
    - Rotational equilibrium is where something is at rest or rotating at a constant angular speed.
  - Since there is no change in the velocity no force is acting on the object.
    - This in many cases is the resultant force (sum of multiple forces).
      - If they're in opposite directions they would cancel out. Seen in the images below:
        - 

Figure 6 Two equal forces in different directions.

      - The forces usually are equal in size and opposite in direction, they will cancel out and be in equilibrium.
      - If they have the same magnitude but aren't in opposite direction then equilibrium can't be achieved.
        - Photo 6 shows two objects which have opposite and equal forces in the horizontal plane, meaning they'll cancel out in the horizontal plane, but they're in the same vertical plane meaning that they will add together in the vertical plane.
        - This unbalance in the vertical plane will result in acceleration in the vertical axis.
      - For the free-body diagram of a ring, the reason that its at equilibrium is because of the fact that the vector sum of all of the 3 forces add up to zero.
        - Free body diagram:
          -  Balanced diagram



- The diagram shows that the arrows can be moved to form a closed triangle (Black arrows). this is known as the triangle of forces.
-

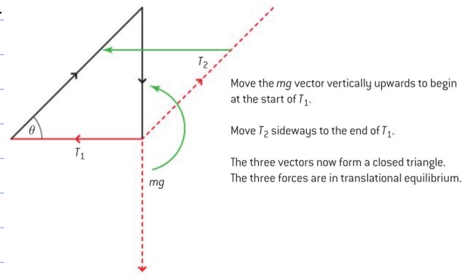
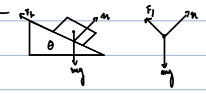


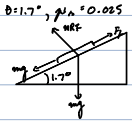
Figure 8 Making a triangle for forces.

- Algebraically this must be true because:
  - $T_1 = T_2 \cos\theta$ (horizontally) and $mg = T_2 \sin\theta$ (vertically)
  - so $T_1^2 = T_2^2 \cos^2\theta$ and $(mg)^2 = T_2^2 \sin^2\theta$
  - $T_1^2 + (mg)^2 = T_2^2(\sin^2\theta + \cos^2\theta)$
    $T_1^2 + mg^2 = T_2^2$
- This shows that a triangle of forces will be at translation equilibrium.

- solid friction
  - Friction is a force that occurs between two surfaces in contact.
  - solid friction has the properties of:
    - the force of friction increasing as you pull harder on a newton spring connected to a mass.
      - Resulting in the object not moving instantly.
    - Eventually if you pull hard enough the object starts moving, at a lower force than the max static value.
      - this new value is maintained as the object moves steadily.
    - Stick slip behavior is when the object moves and then stops, this is due to two different friction values on the surface.
    - Friction force depend on the magnitude of the weights.
  - Friction forces are defined as:
    - Static friction:
      - this is the force that keeps an object at rest.
      - this force must be overcome to make an object move.
      - Its seen when an object is at rest.
    - Dynamic friction:
      - this occurs when two objects are moving relative to each other and rub together.
      - Observed when an object is in motion.
  - As the pulling force increases but the object isn't moving, the friction force is static.
    - Eventually with enough force the static friction will be overcome.
  - As the object starts moving its friction value will drop down to a lower value, known as dynamic friction.
  - static friction
    - The static friction value is given by: $F_f \leq \mu_s R$.
      - F_f is the frictional force exerted by the surface on the object, R is the normal reaction force of the surface on the block.
        - the normal reaction force is the force of the surface on the block. this equals the weight of the object as there is no vertical acceleration.
      - μ_s is the coefficient of static friction.
      - The ≤ symbol shows that the F_f value can have a value that varies, between these limits F_f is equal to the pull on the block.
        - When F_f = applied force, then the object is just about to move.
        - When F_f < applied force, then the object is moving, then dynamic friction occurs.

- Dynamic friction
  - This only applies to objects in motion.
  - The friction drops from its max value, which is given by the static friction.
    - This value depends on the total reaction force and the dynamic friction coefficient.
      - $F_f = \mu_d R$
      - μ_d is the coefficient of dynamic friction.
  - It's possible that the values for μ_d & μ_s are greater than 1, meaning that the value of friction is greater than the weight of the block.
- Friction between a block and a ramp
  - 

    - n (normal reaction force) $= mg\cos\theta$
    - F_f (friction force) $= mg\sin\theta$
    - $\mu_s = \tan\theta = \frac{F_f}{n}$
- Worked example
  - $F = 280N$, $45°$ to floor, $m = 50kg$
    - $(\sin 45°)(280N) = 198N$
    - $mg = 50 \cdot 9.81 = 490.5N$
    - $280\cos 45° = 198N$
    - vertical component $= NRF + URF$
      $= 490.5 + 198N$
      $= 688.5N$
    Friction force $= 198N$

    $\mu_d = \frac{198}{688}$
    $= 0.29$
  - $\theta = 1.7°$, $\mu_s = 0.025$
    

    since $\mu_s = 0.025$, and the calculated value to keep the skiis from sliding away is 0.0297, then the skiis will slide down the ramp.

    $\mu_s = \frac{F_f}{n}$
    $\mu_s = \frac{(mg\sin 1.7)}{(mg\cos 1.7)}$
    $\mu_s = 0.0297$

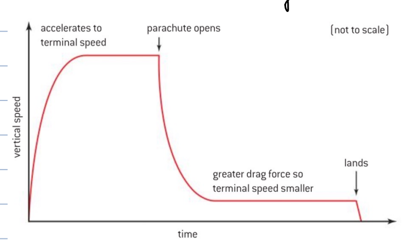
- Origins of friction
  - At the atomic level the surfaces of objects aren't smooth, but instead is jagged.
    - They're full of peaks and troughs.
  - When static friction occurs, the peaks of one object are in the troughs of the other, and enough force has to be applied to break the troughs and move the object.
  - When dynamic friction occurs, the peaks of one object raise up a bit so that less force is required for the object to move.
  - Applying a lubricant on a surface will fill the spaces between the peaks and troughs, reducing contact.
    - Reducing μ_s & μ_d.
- Fluid resistance and terminal speed
  - The drag force is the resistance force caused by the motion of a body through a fluid. E.g water and air.
    - Air resistance is the transfer of energy of the moving body to the medium.
  - Skydiving
    - The air resistance force will act opposite of where the person that is falling / running / etc.
    - The faster the person is moving the greater the air resistance.
      - Therefore the skydiver will start decelerating and eventually falling at a constant rate.
      - This is known as its terminal velocity.
    - 

Figure 12 Speed–time for a parachute jump.

    - Once the parachute opens, the larger surface area will have a greater drag force, greatly slowing down the velocity.
      - This is because the skydiver will accelerate up quickly.
      - Once that has occured, there will be a balance again, and a constant velocity.
  - max speed of a car

- One of the factors that determines the max speed of a car is the amount of power that the engine can exert on the tyres.
- There is also drag due to the air.
    - Which increases as the cars velocity increases.
    - Typically, when the speed doubles the drag force will increase by a factor of four.
- When the max energy output of the engine every second is used to overcome the energy losses (wind drag & friction), then the car can't accelerate anymore and will have reached max speed.

- Worked example (61)
    - $m = 80$ $F = mg$
      $= 80 \cdot 9.81$
      $= 784.8N$ ✓
    - 0N because its constant velocity

- 2.3 Work, energy, and power
    - Equations
        - Work: $W = Fs\cos\theta$
        - Kinetic energy: $E_k = \frac{1}{2}mv^2$
        - Elastic potential energy: $E_p = \frac{1}{2}k(\Delta x)^2$
        - Change in gravitational potential energy: $\Delta E_p = mg\Delta h$
        - power $= Fv$
        - Efficiency $= \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$

    - Energy forms and Transfers
        -

Energy	Nature of energy associated with...	Notes
kinetic	the motion of a mass	$\frac{1}{2}mv^2$
(gravitational) potential	the position of a mass in a gravitational field	sometimes the word "gravitational" is not used = mgh
electric/magnetic	charge flowing	
chemical	atoms and their molecular arrangements	
nuclear	the nucleus of an atom	related to a mass change by $\Delta E = \Delta mc^2$
elastic (potential)	an object being deformed	The word "potential" is not always used
thermal (heat)	a change in temperature or a change of state	A change of state is a change of a substance between phases, i.e. solid to liquid, or liquid to gas. This is referred to as "energy transferred as a result of temperature difference" in line with the IB Guide. The colloquial term "heat" is usually acceptable when referring to situations involving conservation of energy situations.
mass	conversion to binding (nuclear) energy when nuclear changes occur	$E = mc^2 \rightarrow m = \frac{E}{c^2}$
vibration (sound)	mechanical waves in solids, liquids, or gases	the amount of sound energy transferred is almost always negligible when compared with other energy forms
light	photons of light	sometimes called "radiant energy" another form of electric/magnetic $E = hf$

        - One joule is the energy required when a force of a newton acts through a distance of one meter.
        - Principle of conservation of energy, energy can't be created or destroyed.

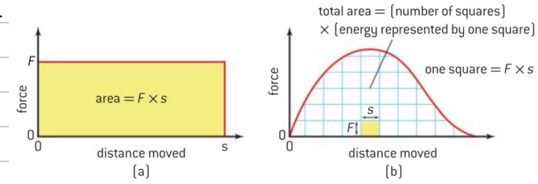
    - Doing Work
        - Work done (J) = force (N) × distance (m)
        - Where something like the wind does work on a sail, which is an angle, the equation becomes: work done $= F\cos\theta \times s$.

    - Work done against a resistive force
        - $W = F \times s \times \cos\theta$ (if at angle)

    - Worked example
        - thrust $= 3.5 \cdot 10^3 N$, $s = 15km$
          $W = (3.5 \cdot 10^3)(15000)$
          $= 5.25 \cdot 10^7 J$
        - $s = 9.5m$, $F = 55N$ at $50°$
          $W = (55\cos(50))(9.5)$
          $W = 301J$
        - $E_k = 24J$ $\frac{24}{4} = s$
          $s = 6m$

    - Force - distance graph
        - For a constant force the graph of force will be a straight line parallel to the x-axis (figure a)
        - The area of the graph represents the work done.

- force - distance graphs:



▲ Figure 4 Force–distance graphs.

- Worked example
  - $W = \frac{1}{2}(5)(4 \cdot 10^{-2})$

    $W = 0.1J$

- Power
  - Example: Jean, 150s: $mg: 650N$, $h: 70m$, $P = \frac{650 \cdot 70}{150} = 303W$
    Philips, 300s: $mg: 650N$, $h: 70m$, $P = \frac{650 \cdot 70}{300} = 151.67W$
  - Power is used to measure the rate of doing work.
    - In other words: the number of joules that can be converted every second.
    - power = $\frac{\text{energy transferred}}{\text{time taken for transfer}} = \frac{W}{t}$
      - $1W \equiv 1Js^{-1}$
  - power = force × speed — Worked example: $P = 20kW$, $v = 25ms^{-1}$, $F = \frac{20000}{25} = 800N$

- kinetic energy
  - kinetic energy is the energy that an object in motion has
  - $E_k = \frac{1}{2}mv^2$ or $E_k = \frac{1}{2}mu^2$ & $\Delta E_k = \frac{1}{2}m(v^2 - u^2)$
  - An object at rest will accelerate to have a final velocity of $10ms^{-1}$.
    - the work done on the mass will be equal to the gain in E_k.
  - Worked example
    - $1700 \, kmh^{-1} = v$, $m = 7800kg$
      - $E_k = \frac{1}{2}(7800)(1700)^2$

        $E_k = 8.70 \cdot 10^8 J$
    - $m = 1.3 \cdot 10^3 kg$, $u = 12ms^{-1}$, $v = 20ms^{-1}$
      - $\Delta E_k = \frac{1}{2}(1.3 \cdot 10^3)(20^2 - u^2)$

        $= 1.7 \cdot 10^5 J$

- gravitational potential energy, GPE
  - GPE is the energy an object has because of its position in a gravitational field.
    - It's given by the equation: $GPE = mgh$
    - A change in the GPE is written as: $\Delta GPE = mg\Delta h$

- Energy moving between GPE and KE
  - When an object is dropped from a point, the gravitational potential energy will be converted to kinetic energy.
    - This is because of the fact that the object will gain velocity, and lose height.
  - $\Delta E_p = mg\Delta h = \frac{1}{2}mv^2$ and $v = \sqrt{2g\Delta h}$
    - E.g. $\Delta h = 50m$: $v = \sqrt{2(9.81)(50)}$

      $= 31.32ms^{-1}$

      $\approx 31.3ms^{-1}$

- Worked example
  - $0.35kg$, $v = 8ms^{-1}$
    - $E_k = \frac{1}{2} 0.35 \cdot 8^2$ - $GPE = 11.2J$

      $= 11.2J$
    - $mgh = 11.2J$

      $h = \frac{11.2}{0.35 \cdot 9.81}$

      $h = 3.3m$
  - $v = \sqrt{2g\Delta h}$

    $= \sqrt{2(9.81)(0.15)}$

    $= 1.72ms^{-1}$

- Elastic potential energy
  - Most springs are designed to store energy when a force is applied to it, and then return it when the force stops.

- the materials that return the energy have stored elastic potential energy. Given by Hooke's law.
- Hooke showed that with a small mass attached to the spring, the extension of the spring will be directly proportional to the load.
  - the graph of force (F) against extension (Δx) is a straight line through the origin.
  - $F \propto \Delta x$ or $F = k\Delta x$, where k is the spring constant.
    - Units = Nm^{-1}.

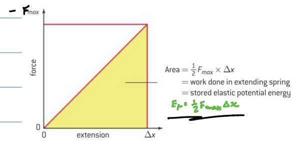


Figure 8 Work done in stretching spring.

- $E_p = \frac{1}{2} k (\Delta x)^2$
  - Potential energy stored equation

- Worked example
  - $k = 48\,Nm^{-1}$, $\Delta x = 0.4\,m$
    - $E_p = \frac{1}{2}(48)(0.4)^2$
      $= 0.38J$
  - $m = 0.78\,kg$, $\Delta x = 60\,mm$, tension of string (F_{max}) $= mg = 0.78 \cdot 9.81 = 7.64N$
    - $E_p = \frac{1}{2} F_{max} \Delta x$
      $= \frac{1}{2}(7.64)(0.060)$
      $= 0.23J$

- Efficiency
  - Energy is always lost in real life.
    - Energy can be lost to heat due to friction or to elastic potential energy when the spring changes shape.
  - Efficiency is used to calculate the energy loss.
    - Efficiency is defined as: How much useful energy comes out compared to the total energy put in.
    - Efficiency $= \frac{\text{useful work out}}{\text{total work in}} = \frac{\text{useful power out}}{\text{total power in}}$

- Worked example
  - $mg = 150N$, $\Delta h = 7.2m$, $E = 3.5 \cdot 10^4 J$
    - $E_p = 150 \cdot 7.2$
      $= 1080J$
    - $\frac{1080}{3.5 \cdot 10^4} = 0.030857$
      $= 3.09\%$

- 2.4 Momentum
  - Equations:
    - Momentum: $p = mv$
    - Newton's second law (momentum version): $F = \frac{\Delta p}{\Delta t}$
    - kinetic energy: $E_k = \frac{p^2}{2m}$
    - impulse: $= F\Delta t = \Delta p$
  - Momentum
    - the product of the mass and the instantaneous velocity is the momentum ($p = mv$).
    - Basic ideas of momentum:
      - momentum is a vector, since it uses velocity which is also a vector quantity.
        - It's in the same direction of the velocity.
      - Units = $kg\,ms^{-1}$
      - A change in velocity will result in a change in momentum.
  - Worked example
    - $m = 0.25kg$, $v = 7.4ms^{-1}$
      - $p = 0.25 \cdot 7.4$
        $= 1.85\,kg\,ms^{-1}$
    - $0.25kg$, $u = 7.4ms^{-1}$, $v = 5.8ms^{-1}$
      - $p_i = 0.25 \cdot 7.4 = 1.85$
        $p_f = 0.25 \cdot 5.8 = 1.45$
      - $\Delta p = p_i + p_f$
        $= 3.3\,kg\,ms^{-1}$
  - Collisions and changing momentum
    - In a newton cradle, when a ball is moved up it will gain gravitational potential energy which is converted to kinetic energy.
      - When the ball hits the other balls, it will compress it slightly, and this compression will travel throughout the balls in the middle.

- when the compression eventually reaches the outmost ball, the elastic potential energy will be converted to kinetic energy which will be converted to gravitational potential energy as the ball moves further up.
- It can also be thought as the transfer of momentum. As the ball moves faster it gains momentum. When it hits the ball next to it, it will transfer momentum to the neighboring ball.
- These interactions are called collisions.
- The momentum of the system is conserved.
  - momentum is always constant if no external force acts on the system.
    - this is known as the principle of conservation of linear momentum.
- Elastic collisions
  - Elastic collisions are: A collision between two objects where the total kinetic energy of the two objects is the same.
    - For it to occur no permanent deformation must occur, and no energy must be lost.

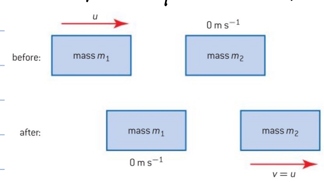


Figure 3 Elastic collision between two identical masses.

- Momentum is conserved because: $m_1 u = m_2 v$
  - The first object transfers all of its momentum to second one.
- Two objects with different masses & no energy lost

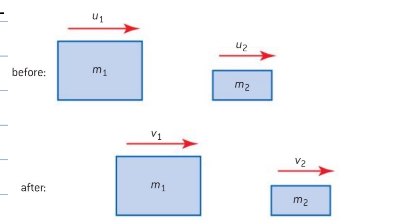


Figure 4 Two moving objects with different mass in an elastic collision.

- $m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$
- kinetic energy is conserved:
  - $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$ and $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1$

- When mass$_1$ is much greater than mass$_2$, the following scenarios occur when they collide and m_1 is stationary.
  - Since the mass of object 2 (m_2) is $\ll m_1$, it will "bounce off" of the m_1 when it collides with it.
    - the m_2 will gain velocity in the opposite direction of its original trajectory, and m_1 will gain some momentum in the original direction.
- When m_1 is much smaller than m_2, m_1 colliding with m_2.
  - The original mass won't lose a lot of its original velocity, the Δp of m_1 will be given to m_2, which moves in the original direction of m_1.
    - m_2 will travel at twice the original velocity of m_1.
- Two objects colliding when energy is lost
  - When a moving object collides with a stationary one, and they stick together, some of the initial kinetic energy is lost.
  - After the collision there is a single object (two objects combined), with a combined mass, and a common velocity.
    - This is an inelastic collision.
  - The momentum is: $m_1 u_1 = (m_1 + m_2) v_1 \rightarrow v_1 = \frac{m_1 u_1}{m_1 + m_2}$
  - kinetic energy is: $E_k = \frac{m_1^2 u_1^2}{2(m_1 + m_2)}$.
    - Ratio: $\frac{m_1 + m_2}{m_2}$
- Two objects when energy is gained
  - A case where two objects gain energy while being initially still is when magnets that have like poles facing one another are placed on the front of the cars.
    - Once they're not being held, they will repel one another, gaing energy.
  - Initially the momentum is 0 since the objects aren't moving.
  - After the collision, $m_1 v_1 + m_2 v_2 = 0 \rightarrow m_1 v_1 = -m_2 v_2$.

- Worked example
  - $m = 4500\,kg$, $u_1 = 1.8\,ms^{-1}$, $u_2 = 0\,ms^{-1}$, $m_2 = 1500\,kg$
    - $m_1 v_1 = p_1$, $p_1 = 8100\,kgms^{-1}$
      $8100 = (m_1 + m_2) v_1$, $v_1 = \frac{8100}{m_1 + m_2} = 1.35\,ms^{-1}$
  - $m_1 = 0.5\,kg$, $u_1 = 3.8\,ms^{-1}$, $u_2 = 0\,ms^{-1}$, $3\,kg = m_2$, $v_2 = 0.65\,ms^{-1}$
    - $p_1 = 0.5 \cdot 3.8 = 1.9\,kgms^{-1}$
      $p_2 = 3 \cdot 0.65 = 1.95$
      $p_2(\text{small}) = m_1 v_1$, $\frac{-0.05}{0.5} = v_1$, $v_1 = -0.1\,ms^{-1}$
  - $u_1 = 2.5\,ms^{-1}$, $u_2 = 0\,ms^{-1}$
    $v_1 =$, $v_2 = 1.9\,ms^{-1}$
    $m_1 = 6000\,kg$, $m_2 = 3000\,kg$
    $m_1 u_1 = p_1$
    $p_1 = (6000)(2.5) = 15000\,kgms^{-1}$
    $p_2 = 0\,kgms^{-1}$, $p_2 = 1.9 \cdot 3000 = 5700$
    $\frac{9300}{6000} = v_2$
    $v_2 = 1.55\,ms^{-1}$ ✓
    Initial $E_k = \frac{1}{2}(6000)(2.5)^2 = 18750\,J$
    Final $E_k = \frac{1}{2}(6000)(1.55)^2 + \frac{1}{2}(3000)(1.9)^2 = 12622.5$
    $\Delta E_k = 6128\,J \approx \underline{6.1 \cdot 10^3\,J}$ ✓

- Energy and momentum
  - $E_k = \frac{p^2}{2m}$

- Applications of momentum conservation
  - Recoil of a gun
    - The initial momentum of the gun and the bullet is zero.
    - When the gas in the chamber behind the bullet explodes, a force is generated on the bullet and the gun.
    - The momentum must be zero, as it initially was zero, therefore, since the bullet gains momentum in the forward direction, the gun must gain momentum in the opposite direction.
  - Water hoses
    - The cross-sectional area of the hose is greater than that of the nozzle.
    - The mass of water that emerges every second is equal to that which flows past a point every second.
      - Therefore, the speed of the water out of the hose must be greater than that in the hose.
      - The water therefore gains momentum as it leaves the hose due to its larger speed.
    - Since momentum has to be conserved, there will be a force backwards
    - The E_k and momentum is being supplied by the water pump that feeds water to the hose.
    - The momentum lost per second is: $m_{\text{water leaving}} \times v_{\text{water leaving}} - v_{\text{in hose}}$.
      - $\Delta p = \frac{\Delta m}{\Delta t}(v - u)$. $\Delta p s^{-1} = \rho A v (v - u)$
    - Mass of water leaving in one second $\frac{\Delta m}{\Delta t}$: $\frac{\Delta m}{\Delta t}$ = density of water × volume of water $= \rho A v$.
    - The force on the hose is given by: $F = \frac{\Delta m}{\Delta t} \times \Delta v$.
    - Worked example
      - $0.48\,kg\,s^{-1}$, $A = 8.4 \cdot 10^{-5}\,m^2$, $u = 0.71\,ms^{-1}$, $\rho = 1000\,kg\,m^{-3}$
        - $0.48 = 1000 \times v \times 8.4 \cdot 10^{-5}$
          $v = \frac{0.48}{1000 \cdot 8.4 \cdot 10^{-5}}$
          $v = 5.71\,ms^{-1}$ ✓
        - $F = \frac{\Delta m}{\Delta t} \times \Delta v = \underline{2.4\,N}$

- Impulse
  - Δp = force · time
  - The product of force · time (Δp) is the impulse.
    - Units = Ns.
  - Worked example:
    - $\Delta p = 85\,Ns$, $m = 5\,kg$, $t = 2\,s$
      - $\Delta p = kg\,ms^{-1}$ $\frac{85}{5} = 17\,ms^{-1}$ 2 second $= \underline{34\,m}$

- Force - Time graph
  - In a force-time graph, the area under the curve represents the change in momentum.

- Worked example
  - $\Delta p = \frac{1}{2} 5 \cdot 15 + 15 \cdot 15 + \frac{1}{2} 5 \cdot 15$
    $\Delta p = \underline{300\,kg\,ms^{-1}}$
  - $\frac{200}{10} = \underline{20\,N}$
    - $\Delta E_k = \frac{1}{2} m (v^2 - u^2)$ or $\Delta E_k = \underline{p^2}$

$300 = mv$

$v = 6\,ms^{-1}$

$= \frac{1}{2}(40)(25)$

$\Delta E_k = 500J$

$2m$

$= \frac{200^2}{2(40)}$

$= 500J$

- **Rewriting Newton's second law**
  - $F = \frac{\Delta mv}{\Delta t} = \frac{\Delta p}{\Delta t} \longrightarrow F = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{t}$

- **Worked example**
  - $35g$, $3.5\,gs^{-1}$, $v = 130\,ms^{-1}$

    $a = \frac{\Delta v}{\Delta t} = \frac{-v\Delta m}{m\Delta t}$

    $= \frac{130 \cdot 3.5}{65}$

    $= 7\,ms^{-2}$

- **Rockets**
  - Rockets always lose mass in the form of fuel, meaning m isn't constant.
  - $F = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t} \rightarrow \frac{m\Delta v}{\Delta t} = -\frac{v\Delta m}{\Delta t}$ ($F = 0$ as there is no external force)
    - $\frac{m\Delta v}{\Delta t}$ refers to the instantaneous mass, and $-\frac{v\Delta m}{\Delta t}$ is the ejection speed of the fuel.
    - meaning acceleration is: $a = \frac{\Delta v}{\Delta t} = -\frac{v\Delta m}{m\Delta t}$.

- **Helicopters**
  - $mg = \frac{m\Delta v}{\Delta t} + \frac{v\Delta m}{\Delta t}$
    - When hovering, $\Delta v = 0$ so $mg = \frac{v\Delta m}{\Delta t}$

- **Momentum & safety**
  - Increasing the time will mean that the force experienced will be lower, that's why seat belts exist.
    - $F = \frac{\Delta p}{t}$

- **Sports**
  - Increasing contact time between the person and the object will mean that the impulse (Δp) will be larger.